# Fundamentals of Heat Transfer

## RADIATION

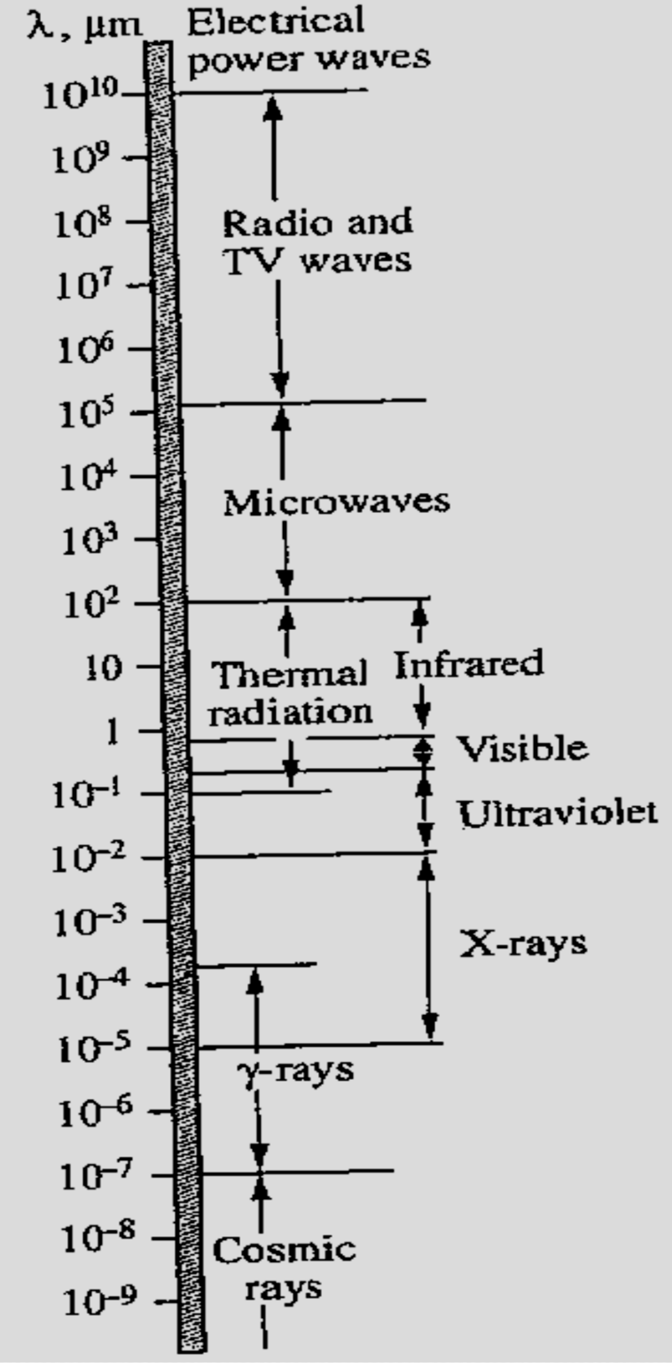
# INTRODUCTION

What is radiation?

- Radiation is the energy emitted by matter in the form electromagnetic waves or protons as a result of the changes in the electronic configuration of the atoms or molecules because of their temperature.
- All bodies at a temperature above absolute zero emit thermal radiation.
- **Thermal Radiation occurs approximately in the range of 0.1 μm to 100 μm , including the visible and the near, middle and far IR (infrared) regions**

# INTRODUCTION

Electromagenetic wave spectrum




Thermal Radiation occurs approximately in the range of 0.1 μm to 100 μm , including the visible and the near, middle and far IR (infrared) regions

# Radiation depends on :

- Surface temperature
- Surface Radiation properties
- Geometry
- Radiation properties of the medium

In general thermal radiation has both directional (*depends on the angle relative to the surface*) and spectral (*depends on wave length*) dependence. In this course surfaces are taken to be diffuse and consider only total emission

# Black Body (Ideal Surface) :

- Absorbs all incident radiation regardless of wavelength (λ) or **angle of incidence** and **reflect zero radiation**. It is a diffuse surface emitter (NO preferred direction for emission)
- Radiation emitted by a black body per unit time and per unit surface area was determined experimentally by Joseph Stefan in 1879 → Stefan's law

# Radiation

- **Radiation** is the energy emitted by matter in the form of *electromagnetic waves* (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules.
- Unlike conduction and convection, the transfer of energy by radiation does not require the presence of an *intervening medium.*
- We focus on thermal radiation
- All bodies at a temperature above absolute zero emit thermal radiation.

- The maximum rate of radiation that can be emitted from a surface at an absolute temperature *Ts* (in K or R) is given by the **Stefan–Boltzmann law** as:

$$\dot{Q}_{emit} = \varepsilon\sigma A_s T_s^4 \qquad (W)$$

- For 2 bodies at temps:

$$\dot{Q}_{rad} = \varepsilon\sigma A_s (T_s^4 - T_{surr}^4) \qquad (W)$$

# Laws of Thermal Radiation

- **Spectral black body emissive power**

  Radiant Energy emitted by a black body at T per unit area of surface per unit wavelength (Planck's Law):

$$E_{b_\lambda} = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1} \quad \ldots\ldots\ldots\ldots(1)$$

where

$C_1 (=2\pi h c_o) = 3.742 \times 10^8$ W(μm)$^4$/m$^2$; $C_2 (= h c_o/k) = 1.4389 \times 10^4$ (μm.K);

$k = 1.38065 \times 10^{-23}$ J/K is the Boltzmann constant; $h = 6.6256 \times 10^{-34}$ J.s is the universal Planck constant; $c_o = 2.998 \times 10^8$ m/s is the speed of light in vacuum

- Equation (1) is valid for a surface in vacuum or gas. For other media, $C_1$ is replaced by $C_1/n^2$, where n is the index of refraction of the medium

# Laws of Thermal Radiation

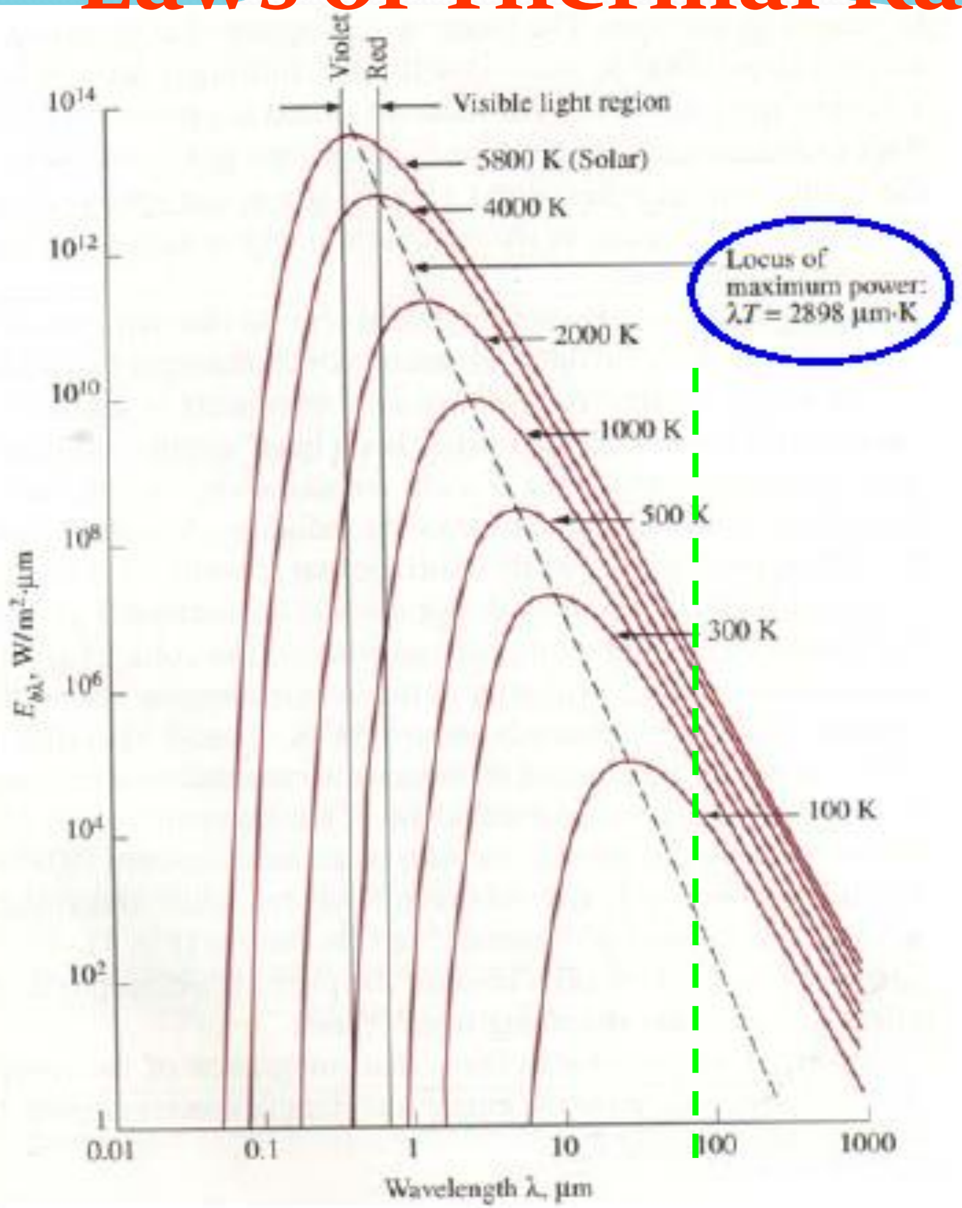



Figure 1: Emissive power vs λ at diff. temp

1. The emitted radiation is a continuous function of time. At any specific temperature, it increases with λ , reaches a peak, and then decreases with increasing λ
2. At any λ, the amount of emitted radiation increases with increasing temperature
3. As temperature increases , the curves shift to the left of the shorter λ
4. The radiation emitted by the sun which is considered as a black body at 5800K, reaches its peak at visible region.

## Wien’s displacement law

The wavelength corresponding to the peak radiant flux is inversely proportional to temperature

$$(\lambda T)_{\max\ power} = 2897.8\,\mu m.K \ldots\ldots\ldots\ldots(2)$$

If you can integrate spectral blackbody emissive power over the wavelength spectrum can give total black body emissive power

$$E_b = \int_0^{\infty} E_{b_\lambda} \, d\lambda$$

$$E_{b_\lambda} = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1}$$

$$E_{b_\lambda} = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1} \Rightarrow \frac{E_{b_\lambda}}{T^5} = \frac{C_1}{(\lambda T)^5 \left[ e^{C_2/\lambda T} - 1 \right]}$$

$$\frac{E_b}{T^5} = C_1 \int_0^\infty \frac{d\lambda}{(\lambda T)^5 \left[e^{C_2/\lambda T} - 1\right]}$$

Let, $$x = \frac{1}{\lambda T}, \Rightarrow dx = -\frac{1}{\lambda^2 T}$$

$$d\lambda = \frac{(\lambda T)^2}{T} dx \Rightarrow d\lambda = \frac{1}{x^2 T} dx$$

$$\frac{E_b}{T^5} = \frac{C_1}{T} \int_0^\infty \frac{x^3 dx}{e^{C_2 x} - 1}$$

$$\frac{E_b}{T^4} = C_1 \int_0^{\infty} x^3 \left[1 - e^{C^{2x}}\right]^{-1} dx$$

$$= C_1 \int_0^{\infty} x^3 \left[1 + e^{C_2 x} + 1/2e^{2C_2 x} + \ldots\right] dx$$

**Integrate by parts :**

$$\frac{E_b}{T^4} = \frac{\sigma C_1}{C_2^4}\left[1 + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\right]$$

$$= \frac{\sigma C_1}{C_2^4} \frac{\pi^4}{90}$$

# Laws of Thermal Radiation

**The Stephan-Boltzmann Law**

Thus, Integrating Eq. (1) over all wavelengths to give the heat transferred by radiation from a black body we get

$$E_b = \sigma T^4 \left( \frac{W}{m^2} \right) \ldots\ldots\ldots\ldots\ldots (3)$$

where σ is Stephan-Boltzmann constant (=$5.67\times 10^{-8}$ $W/m^2 . K^4$), T is the absolute temperature of the surface in K

# Laws of Thermal Radiation

For two black bodies at temperatures $T_1$ and $T_2$, the maximum relative interchange between them is given by

$$E_{b,1} = \sigma T_1^{\,4} \ldots\ldots\ldots\ldots\ldots (4a)$$

$$E_{b,2} = \sigma T_2^{\,4} \ldots\ldots\ldots\ldots\ldots (4b)$$

$$E_{b,1-2} = \sigma\left(T_1^{\,4} - T_2^{\,4}\right) \ldots\ldots\ldots\ldots\ldots (4c)$$

# Radiation properties

Emissivity(ε)

Is the ratio of the radiation emitted by the surface at a given temp. to the radiation emitted by a blackbody at the same temp. It is a measure of how closely a surface approximates a black (ε =1)

$$0 \le \varepsilon \le 1 \qquad (5)$$

Total hemispherical emissivity (or "average emissivity")

Is the ratio of radiation emitted surface to the radiation emitted by blackbody

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} \qquad (6)$$

Spectral directional emissivity

The ratio of intensity of radiation emitted by the surface at a specific wave length in specific direction to the intensity of radiation emitted by blackbody at the same temp. at the same wavelength.

# Absorptivity (α), Reflectivity (ρ) & Transmissivity (τ)

Not all surfaces are ideal emitters and absorbers, they are not "Black" but "Grey"

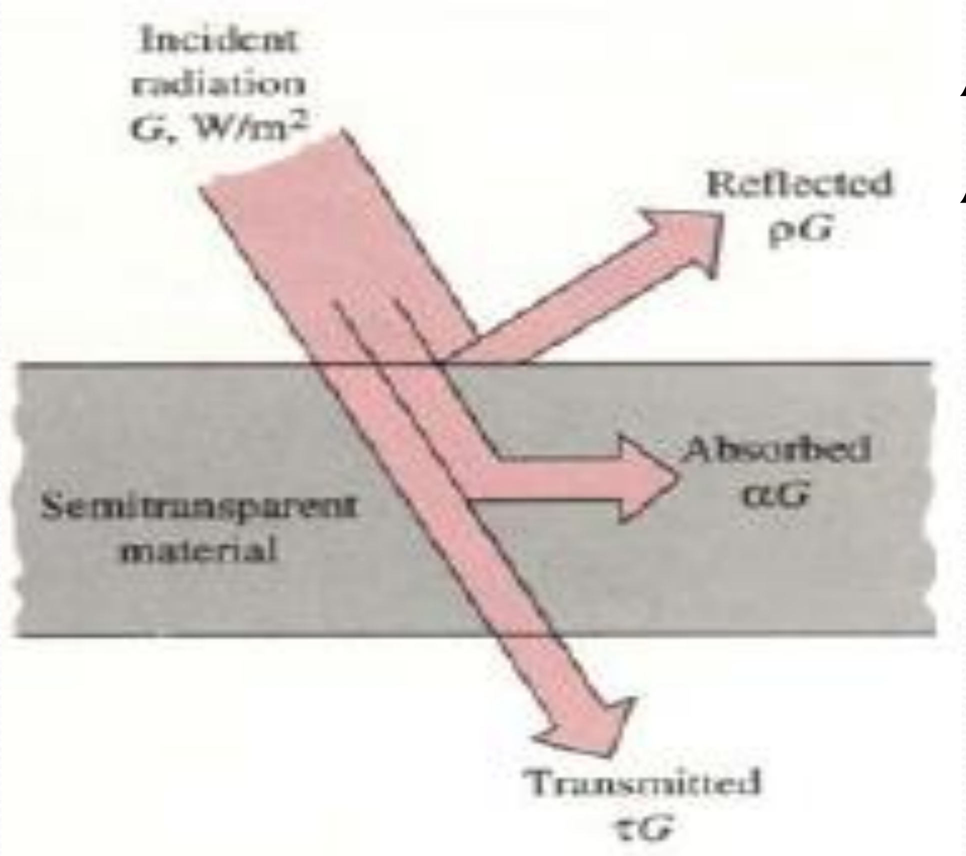



Fig. 6: Radiation incidence

$$\rho G + \varepsilon G + \tau G = G \qquad (7)$$

$$\rho + \varepsilon + \tau = 1 \qquad (8)$$

Most solids except glass and some plastics are opaque, thus $\tau = 0$

# Absorptivity vs. Emissivity

- Kirchhoff's law: emissivity and absorptivity are equal when thermal equilibrium exists (i.e., surface temperature = source temperature of incident radiation);
- In practice, the emissivity and absorptivity of gray surfaces are equal even when thermal equilibrium does not exist — This is experimentally observed fact rather than a universal rule! This rule must not be applied to non-gray real surfaces!!

# Absorptivity vs. Emissivity

- Gray and Diffuse assumption — very commonly used in radiation calculations
- Radiation calculation is quite complex: one of the reasons is the wavelength and direction dependence of radiation properties, e.g.;
  - Diffuse surface: its radiation properties are independent of direction;
  - Gray surface: its radiation properties are independent of wavelength

$$\varepsilon(T) = \varepsilon_{\lambda}(T) = \varepsilon_{\hat{s}}(T) = \text{constant} \ldots\ldots\ldots\ldots(9)$$

$$\alpha(T) = \alpha_{\hat{s}}(T) = \text{constant} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

# Absorptivity vs. Emissivity

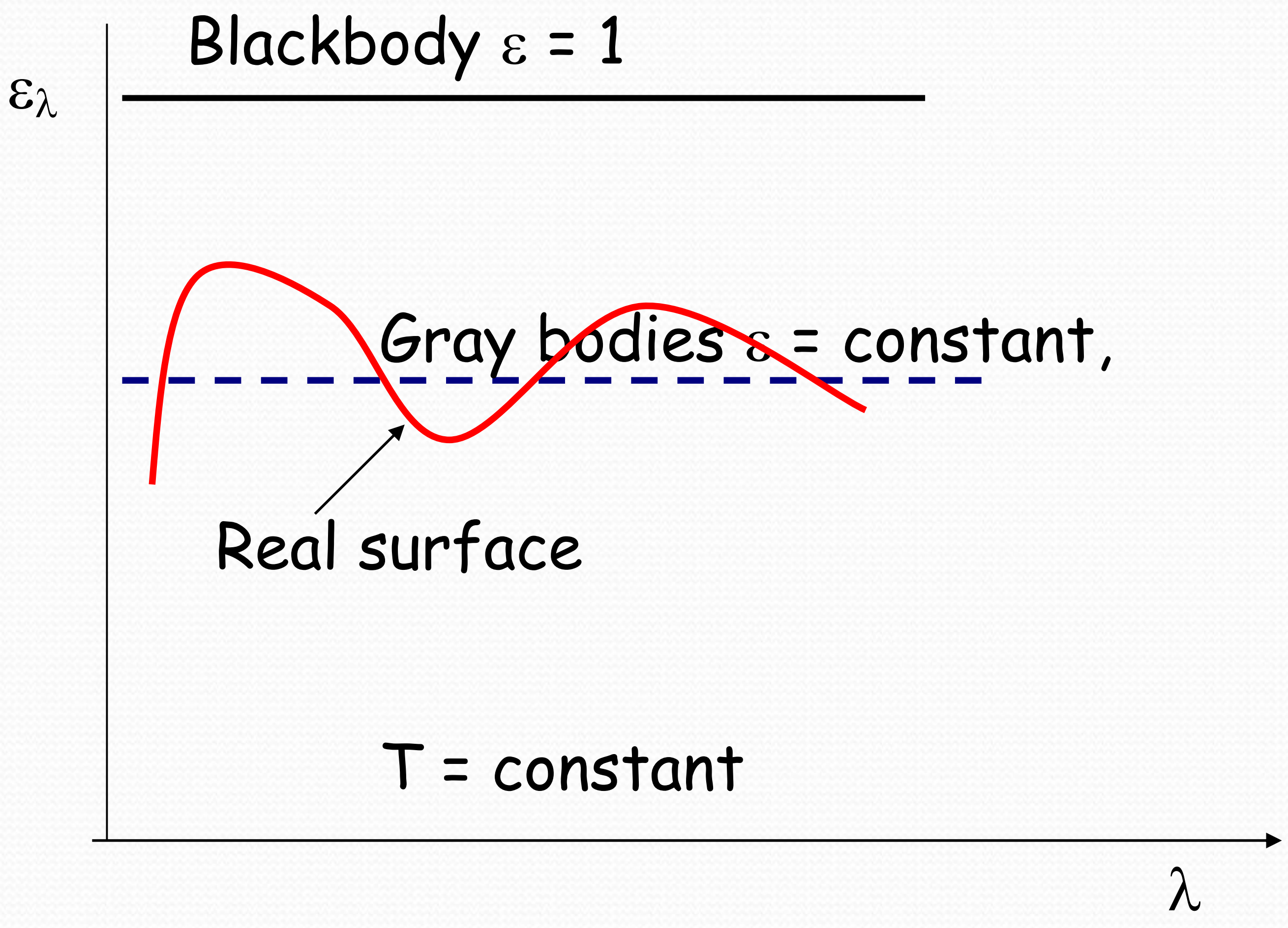
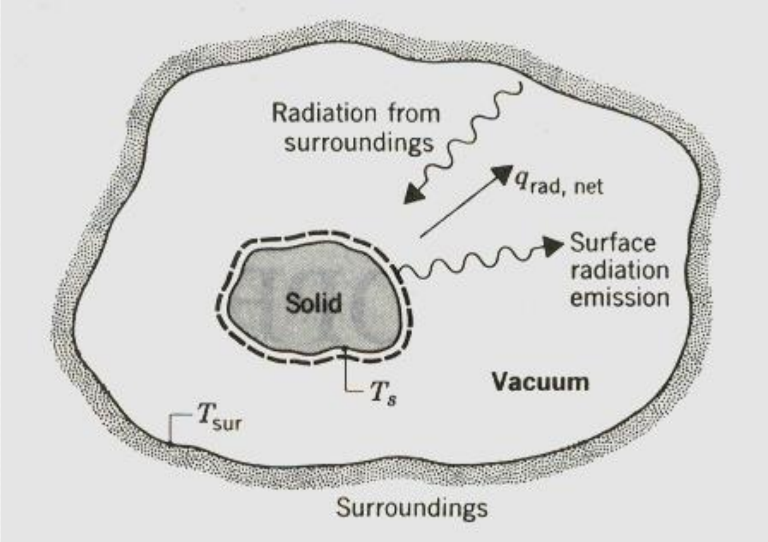



Fig. 7: Variation of Emissivity with wavelength

# Kirchhoff's Law

- When two bodies A and B of areas A1 and A2 are in a large isothermal enclosure:




- Energy absorbed by A from the enclosure

$$A_1 \alpha_1 I_1 ..........(11)$$

- Energy given out by A:

$$E_1 I_1 ..........(12)$$

# Kirchhoff's Law

- At equilibrium these quantities will be equal. In the same way the energy emitted by B will be equal to the energy received

$$A_1 \cdot \alpha_1 \cdot I_1 = A_1 \cdot E_1 \ldots\ldots\ldots\ldots(13)$$

and

$$A_2 \cdot \alpha_2 \cdot I_2 = A_2 \cdot E_2 \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

- But $I_1 = I_2$, giving

$$I = \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E}{\alpha} \ldots\ldots\ldots\ldots(15)$$

- Equation (15) is general for any body and is termed Kirchhoff's law

# Kirchhoff's Law

- Emissivity of a body is defined as the ratio of its emissive power to that of a black body – Eq. (6)

- Since $E/\alpha$ is constant for all bodies, then

$$\frac{E}{\alpha} = \frac{E_b}{\alpha_b} \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

and

$$\varepsilon = \frac{E}{E_b} = \frac{\alpha}{\alpha_b} = \frac{\alpha}{1} = \alpha \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

# Heat Transfer by Radiation

- A body of emissivity ε, at an absolute temperature $T_1$ emits energy $\varepsilon\sigma T_1^4$ per unit area.
- If the surroundings are black, they reflect back none of this radiation, but if they are at an absolute temperature $T_2$ they will emit radiation $\sigma T_2^4$
- If the body is grey it will absorb a fraction ε, so that the net radiation per unit area from the grey body will be

$$\dot{q} = \varepsilon \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) \quad \text{..........(18)}$$

# Heat Transfer by Radiation

- For a material that does not behave as a grey body but as a **reflective emitter**, the absorptivity of the surface at $T_1$ for radiation from surroundings at $T_2$ will be $\alpha T_2$.
- This will not be equal to its emissivity at $T_1$ ($\varepsilon T_1$) but to its emissivity at $T_2$ ($\varepsilon T_2$).
- Under these conditions the general equation for the net exchange of heat becomes:

$$\dot{q} = \sigma \cdot \left(\varepsilon_{T_1} \cdot T_1^{\,4} - \varepsilon_{T_2} \cdot T_2^{\,4}\right) \ldots\ldots\ldots\ldots (19)$$